Rough-Wall Turbulent Heat Transfer with Variable Velocity, Wall Temperature, and Blowing

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An integral prediction method is presented which accurately describes Stanton number behavior for a fully rough turbulent boundary layer flowing over a uniformly rough surface. The kernel function which represents the response of such a system to an unheated starting length is shown to also describe the response to variations in freestream velocity, surface temperature, and blowing. Predictions are compared with experimental data for cases of variable wall temperature, favorable pressure gradients, and variable blowing. Agreement is excellent in all cases.

Nomenclature

c_p = specific heat at constant pressure
F = blowing parameter = $(\rho_0 v_0)/(\rho_\infty U_\infty)$
l = x position of boundary condition step
$\dot{q}''$ = wall heat flux
r = radius of spheres comprising test surface
St = Stanton number = $(\dot{q}'')/(\rho_\infty U_\infty c_p \Delta T)$
St_0 = Stanton number for constant U_∞, ΔT, and F
T = temperature, F
ΔT = wall to total freestream temperature difference, F
U = longitudinal mean velocity
v_0 = velocity of transpired air at wall
x = longitudinal distance measured from test section inlet
Δ_2 = enthalpy thickness
ρ = air density
ξ = dummy integration variable

Subscripts

l = x position where unheated starting length ends
w = wall value
x = at x position of interest
∞ = freestream value
$\infty,0$ = freestream value at test section inlet

Introduction

THE effects of a rough surface on the fluid dynamics and heat transfer in turbulent flows have been a subject of increasing interest in the past few years due to their importance in the design of re-entry vehicles, nozzles, and turbine blades. The flowfield in these applications is typically influenced not only by surface roughness but also by variable freestream velocity, variable wall temperature, and variable blowing rates (as a result of ablation or injection of a coolant). The interaction of these effects with surface roughness has been investigated individually and modeled by several prior studies, and prediction schemes have been offered by several authors.[1-4] The data base which supports these prediction methods is small. There are relatively few studies of heat transfer with roughness and very few with variable boundary conditions. There are studies of nozzle flows by Reshotko et al.[5] and by Banerian and McKillop[2] and results for roughened hemispheres cited by Chen.[3] Reshotko et al.[5] measured heat-transfer distributions in a 1.5-in. throat diameter nozzle roughened by sandblasting, while Banerian and McKillop[2] reported data from a 1-in. throat diameter nozzle roughened with concentric ribs. Both of these investigations were conducted with compressible flow and relatively small models. Chen[3] cited heat-transfer results from supersonic tests on 3.5-in. radius hemispheres with two forms of roughness—one a rectangular pattern of grooves, the other a sand-grain-type finish. To the knowledge of the authors, no data for rough surfaces with variable blowing have been previously reported.

An experimental study of the effects of roughness on turbulent boundary-layer heat transfer has been in progress at Stanford for the past several years. The present data are part of this program, and are concerned with some of the effects of acceleration on rough-wall heat transfer with blowing and variable wall temperature. In the course of this study it was found that the fully rough state of a turbulent boundary layer is markedly different from the smooth state in its response to acceleration. Acceleration was shown to cause an increase in rough-wall Stanton number rather than a decrease, as observed in a smooth-wall situation. The fact that an increase in either the wall temperature or the freestream velocity resulted in an increase in Stanton number suggested that both could be treated in the same way.

The variable wall temperature case for smooth walls was dealt with by Reynolds et al.[6] using superposition based on a kernel function describing the downstream effects of a step change in wall temperature. Whitten[7] extended this to include variable blowing and Orlando[8] used this same method for variable wall temperatures in adverse pressure gradients. However, in Orlando's case it was shown that smooth-wall Stanton number was insensitive to the adverse pressure gradients investigated. Thus, the effects of variable velocity were negligible in his prediction technique.

The energy integral equation for variable velocity, variable wall temperature (but constant fluid properties) can be written

Presented as Paper 76-146 at the AIAA 14th Aerospace Sciences Meeting, Washington, D.C., Jan. 26-28, 1976; submitted May 31, 1977; revision received Aug. 25, 1977.

Index category: Boundary Layers and Convective Heat Transfer—Turbulent.

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as

$$St+F=\frac{1}{U_\infty \Delta T}\frac{d}{dx}(\Delta_2 U_\infty \Delta T) \quad (1)$$

The fact that $(U_\infty \Delta T)$ occurs as a product form, coupled with the experimental observation by Coleman[9] that increases in U_∞ had the same effect as increases in ΔT, suggested the use of the same form of kernel function to account for both effects.

It should be noted that smooth-wall boundary layers do not offer this opportunity. In one sense, the effect of acceleration on a smooth-wall boundary layer is more complex: for smooth-wall flows, there is a threshold value of acceleration beneath which there is little or no effect on Stanton number,[10] and then acceleration drives Stanton number down, not up.

The data in this study were obtained with air as the working fluid. The results and conclusions of this study, therefore, apply to a fluid with a Prandtl number approximately that of air.

Experimental Apparatus

Detailed descriptions of the experimental apparatus and qualification tests have been reported by Healzer[4] and Pimenta.[11] A brief description will be given. The experimental apparatus is a closed-loop wind tunnel using air as both the primary and transpiration fluids. Air temperature is controlled using water-cooled heat exchangers in both the primary and transpiration loops. The 8-ft long, 20-in. wide test section is 4-in. high at its entrance. A flexible plexiglas upper wall can be adjusted to give the desired variation in U_∞.

The test surface consists of 24 plates each 4 in. in the axial direction. The plates (Fig. 1) are 0.5-in. thick and uniformly porous. They are constructed of 11 layers of 0.050-in.-diam OFHC copper spheres packed in the most dense array and brazed together. Each plate has individual electrical power and transpiration air controls and thermocouples for determining plate temperature. Stanton number is determined by subtracting the plate losses (known from energy balance qualification tests) from the measured power input. Uncertainty of the St data is ± 0.0001 Stanton number units (i.e., if $St=0.00200$, the uncertainty is $\pm 5\%$).

The Stanton number data reported here were taken with a wall-to-freestream temperature difference of approximately 30°F to maintain a constant property boundary layer. The freestream velocity at the test section inlet was a nominal 88 ft/sec and all data were taken with a trip installed 3-in. inside the nozzle with the exception of the $F=0.002$, constant U_∞ and T_w case, which was untripped. The turbulent boundary layer was in a fully rough state for all cases reported.

Prediction Method

Method for Variable U_∞, T_w with No Steps in F

It was shown by Healzer[4] and Pimenta[11] that for a fully rough turbulent boundary-layer flow on the present surface with constant U_∞, T_w, and F

$$St=f(\Delta_2/r, F) \quad (2)$$

where r is the radius of the spheres comprising the test surface. Thus, there is a unique curve of St vs x/r for each value of F with constant U_∞ and T_w. The present data for $U_\infty=88$ ft/sec and ΔT constant confirm this, being well represented by the interpolation expression

$$\log_{10}St_0=A+B\log_{10}(x/r)+C\log_{10}^2(x/r) \quad (3)$$

where

$$A=-1.36+48.2F$$
$$B=-0.61-57.4F$$
$$C=0.0675+3.69F$$

for $0\leq F\leq 0.004$. Figure 2 shows the St_0 data and interpolation curves from Eq. (3) for three values of F.

Now consider the case of an unheated starting length with constant U_∞ and F. Reynolds et al.[6] showed that for turbulent flow over a smooth wall with $F=0$, the Stanton number downstream of a step increase in wall temperature could be predicted by

$$\frac{St}{St_0}=\left[1-\left(\frac{l}{x}\right)^m\right]^n \quad x>l \quad (4)$$

where l is the x position of the temperature step, $m=9/10$ and $n=-1/9$. This expression was then used as the kernel function in a superposition integral for predictions with arbitrarily varying ΔT as follows:

$$\frac{St_x}{St_0}=\frac{1}{\Delta T_x}\int_0^x\left[1-\left(\frac{\xi}{x}\right)^m\right]^n\frac{d[\Delta T(\xi)]}{d\xi}d\xi \quad (5)$$

Consideration of a kernel function of the same type for the present rough wall unheated starting length results shows that the data are well represented with $m=1$ and $n=-0.22$. Thus, for fully rough turbulent flow with unheated starting length and U_∞ and ΔT constant

$$\frac{St_x}{St_0}=\left[1-\left(\frac{l}{x}\right)\right]^{-0.22} \quad x>l \quad (6)$$

where St_0 is evaluated at the same F and x as St_x. Figure 3 shows the unheated starting length St data and curves evaluated using Eq. (6) for two cases with $F=0$ and one case with $F=0.0039$.

Coleman[9] showed that for a fully rough turbulent boundary layer, a positive value of dU_∞/dx gave a higher Stanton

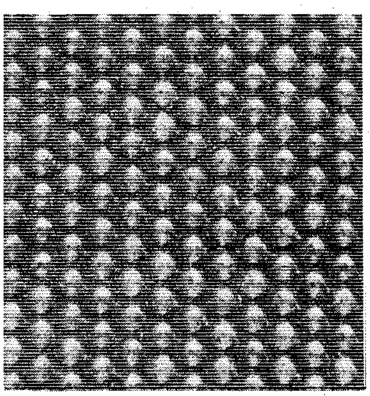

Fig. 1 Closeup photograph of rough test surface.

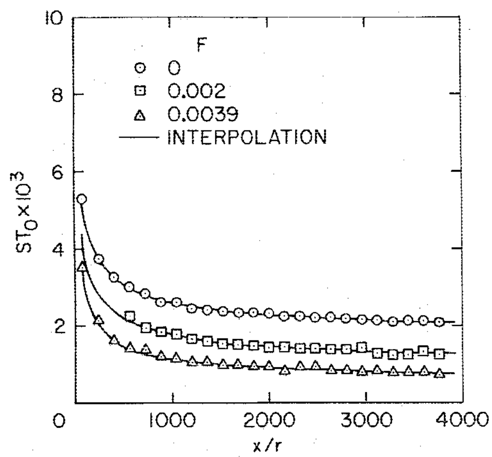

Fig. 2 Baseline St data and interpolations using Eq. (3).

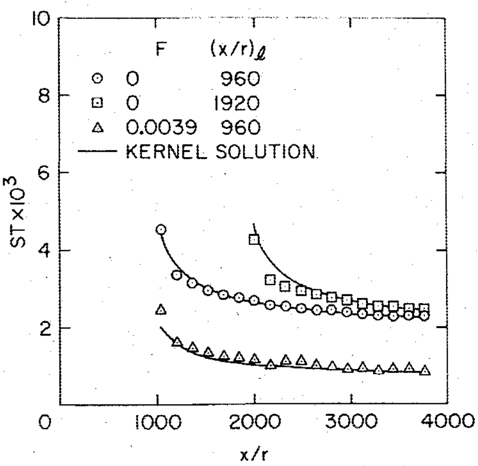

Fig. 3 Unheated starting length St data and kernel solution results.

number than the constant velocity case. Thus, the response of Stanton number to positive dU_∞/dx is the same as to positive values of dT_w/dx. Also, consideration of the energy integral equation (with constant properties) in the form

$$St+F=\frac{1}{U_\infty \Delta T}\frac{d}{dx}(\Delta_2 U_\infty \Delta T) \tag{7}$$

shows that the variables U_∞ and ΔT always appear in the product form as $(U_\infty \Delta T)$. [Of course, Eq. (7) is a conservation equation obeyed by both smooth and rough wall flows. The different response of smooth and fully rough layers to positive dU_∞/dx is evidently related to the effect of the viscous sublayer present in smooth-wall layers on the rate equations.]

These observations lead to the following proposed prediction method for cases of variable U_∞, ΔT, and F in a fully rough flow:

$$\frac{St_x}{St_0}=\frac{1}{(U_\infty \Delta T)_x}\int_0^x \frac{1}{(1-\xi/x)^{0.22}}\frac{d}{d\xi}[U_\infty(\xi)\Delta T(\xi)]d\xi \quad x>\xi \tag{8}$$

where St_0 is evaluated at the same x and F as St_x. The predictions presented in a later section were obtained by numerically integrating Eq. (8). For these calculations, the integral was expanded to a sum of two integrals and the $dU_\infty(\xi)$ term was approximated by assuming a linear variation of U_∞ with ξ across each integration step.

Modification to Incude Steps in F

It was found that Eq. (8) works well except in cases where there are steps in F. A modification of the method is required to account for the "lag" in St after such a step. A simple and satisfactory approach is to define a new St_0 which is modified by the kernel function, giving

$$St_0^*=St_0-\sum_{i=1}^{N}\Delta St_{0_i}\left[1-\left(1-\frac{l_i}{x}\right)^{0.22}\right] \quad x>l_i \tag{9}$$

where $\Delta St_{0_i}=St_0(l_i^+)-St_0(l_i^-)$, and l_i is the position of the ith step. Thus, when there are steps in F, use of St_0^* in place of St_0 in Eq. (8) accounts for the "lag" in St caused by the step. Note that in the absence of steps in F, $St_0^*=St_0$. The present apparatus permits changes in F on 4-in.-wide strips. A monotonic sequence of small steps is functionally equivalent to a ramp in the context of the present procedure.

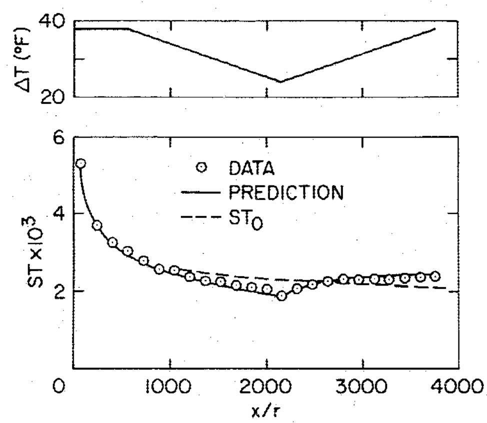

Fig. 4 Comparison of data and prediction—U_∞ constant, $F=0$, bilinear ΔT variation.

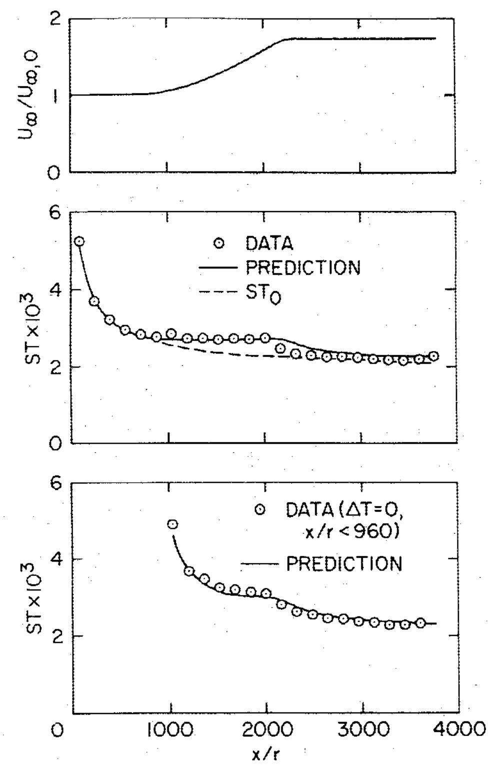

Fig. 5 Comparison of data and prediction—variable U_∞, $F=0$.

Results—Data and Predictions

The experimental Stanton number data are presented in this section and compared with the results of the prediction method discussed earlier. The Stanton numbers are plotted in all cases vs axial distance x normalized by r, the radius of the spheres which comprise the rough test surface.

Figure 4 presents results for a case with constant U_∞, $F=0$, and with ΔT first decreasing linearly, followed by a linear increase. The St data show a decrease below St_0 in the region of negative $d\Delta T/dx$, then an immediate increase when $d\Delta T/dx$ becomes positive. Prediction agrees well with the experimental data. Figure 5 presents results for an accelerated flow case with $F=0$. Two sets of results are shown—one for ΔT constant over the entire test section, the other for an unheated starting length. Stanton numbers are higher than unaccelerated values in the accelerated region, and the data indicate an almost immediate return to the St_0 levels when acceleration is removed. Good agreement between predictions and data is again evident. Figure 6 presents another accelerated case for both $F=0$ and 0.0039 with ΔT constant.

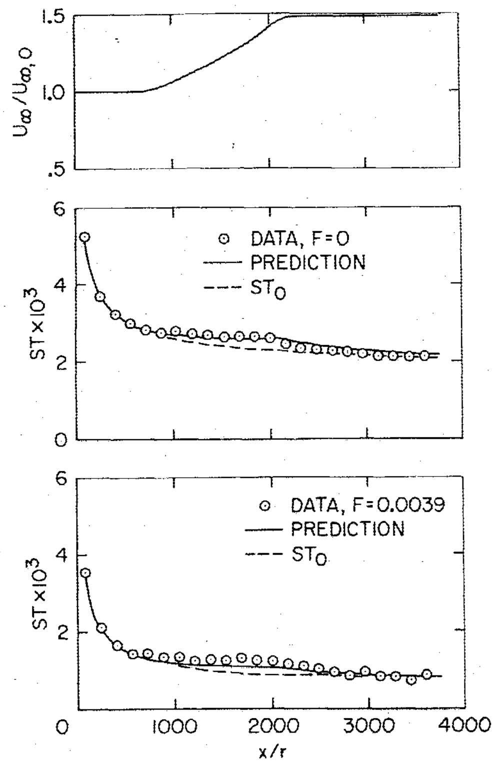

Fig. 6 Comparison of data and prediction—variable U_∞, ΔT constant.

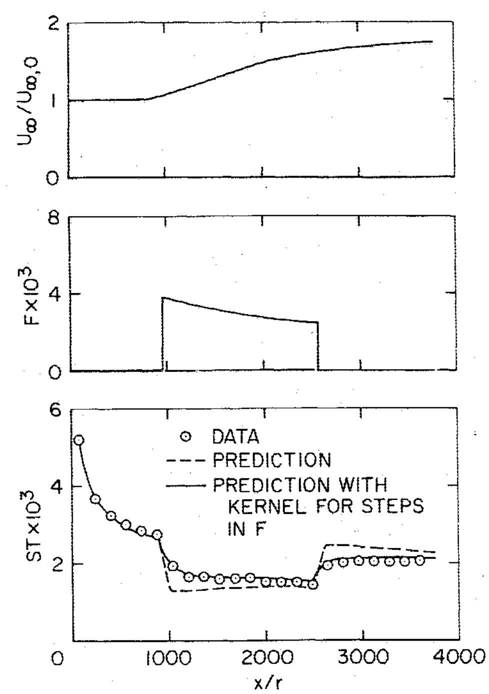

Fig. 7 Comparison of data and prediction—U_∞ variable, ΔT constant, steps in F.

The results show the same trends observed in the previous figure.

Figures 7 and 8 show results for conditions which present the greatest challenge to the prediction method. The flow is accelerated and subjected to a step in F, followed by a variable F and then a step back to $F=0$. Figure 7 presents the ΔT constant case, while Fig. 8 shows results when a step in ΔT is imposed in the region of variable F. In both figures the dashed line is the prediction using Eq. (8), while the solid line shows the prediction including the modification for the steps

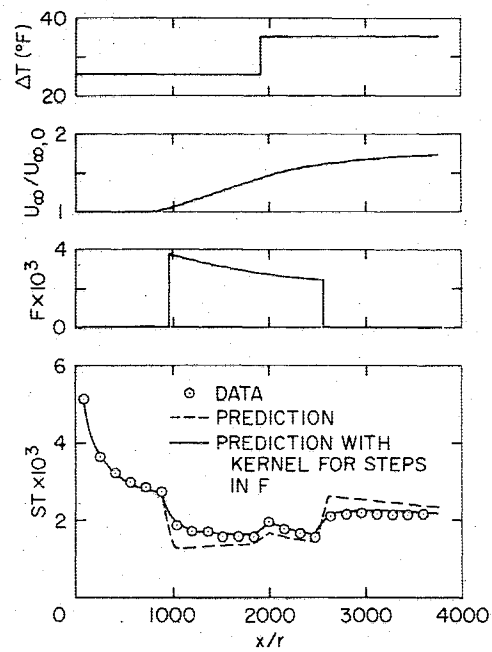

Fig. 8 Comparison of data and prediction—U_∞ variable, steps in ΔT and F.

in F [Eq. (9)]. It is evident that the lag introduced by the steps in F must be taken into account, but once this is done, the prediction method handles the variable blowing very well without further modification.

Summary and Conclusions

Examination of heat-transfer data for accelerating flows on a rough wall with blowing reveals that the effects of acceleration are similar to those of variable wall temperature: a positive gradient in either the freestream velocity or the wall temperature causes an increase in the Stanton number. Taking advantage of this similarity, it is proposed to account for either effect by use of a kernel function similar to that long used in superposition treatment of variable wall temperature. It is shown, by comparison with the data, that the response of Stanton number to variations of ΔT or U_∞ can be represented with good accuracy using this method, even in cases where there is blowing through the wall in the region of acceleration, providing that F is smoothly varying.

The value of Stanton number at a particular x location on the present surface with a fully rough turbulent boundary layer subject to acceleration and variable wall temperature is given by Eq. (8) where St_0 is evaluated at the same x and $F(x)$ as is St_x. If steps in F are encountered, special treatment is required. A method of handling this situation is proposed.

Acknowledgment

This research was supported by the Office of Naval Research, Contract N00014-67-A-0112-0072.

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